A Methodology of Modelling a Wave Power System via an Equivalent RLC Circuit

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Abstract—The equivalent circuit method can be an effective modelling technique for system studies of point absorbing wave energy converters (WECs). For the continuously evolving design and study of WEC systems, an instruction on how to draw the corresponding equivalent RLC circuit model is needed. It is not only vital to make sure the model is correct, but to allow the model to be easily adapted for different cases and implemented by different researchers. This paper presents a force analysis-oriented methodology based on a typical WEC unit composed of a heaving buoy and a linear generator. Three cases are studied in order to demonstrate the procedures: the generator with a retracting spring, the connection line with a rubber damper, and buoy motion in both heave and surge directions. The presented methodology serves as a guide to produce non-linear circuit models that give a reliable description of the dynamics of real wave energy systems.

Index Terms—wave energy, point absorber, system modelling, RLC circuit, Simulink

NOMENCLATURE

\[ g \text{ [m/s}^2] \] Gravitational acceleration  
\[ \rho \text{ [kg/m}^3] \] Water density  
\[ \omega \text{ [rad/s]} \] Angular frequency  
\[ \eta \text{ [m]} \] Surface water elevation  
\[ H \text{ [m]} \] Wave height  
\[ T \text{ [s]} \] Wave period  
\[ H_s \text{ [m]} \] Significant wave height  
\[ T_e \text{ [s]} \] Wave energy period  
\[ f_e \text{ [N]} \] Excitation force per unit length  
\[ F_e \text{ [N]} \] Excitation force  
\[ F_r \text{ [N]} \] Radiation force  
\[ m_a \text{ [kg]} \] Added mass  
\[ m_a(\infty) \text{ [kg]} \] Added mass at infinite frequency  
\[ B \text{ [kg/s]} \] Radiation damping coefficient  
\[ L(t) \text{ [rad]} \] Radiation impulse response function  
\[ S \text{ [m}^2] \] Cross-sectional area of the buoy  
\[ b \text{ [m]} \] Draft of working condition buoy  
\[ b' \text{ [m]} \] Draft of free floating buoy  
\[ V_{sub} \text{ [m}^3] \] Submerged volume of the buoy with draft \( b \)  
\[ V_{sub}' \text{ [m}^3] \] Submerged volume of the buoy with draft \( b' \)  
\[ M \text{ [kg]} \] Mass  
\[ F_{PTO} \text{ [N]} \] Damping force from PTO machine  
\[ F_{line} \text{ [N]} \] Force in the connection line

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I. INTRODUCTION

WAVE energy is a renewable energy source that has potential to significantly contribute to the energy demand in the world. Many different concepts for wave energy conversion have been developed; some let the rising water of waves drive trapped air through a turbine, whereas other converters are oscillating floating body systems, or flapping pendulums at the seabed. This paper concerns point absorbing wave energy converters, based on the device that is developed by the wave energy group at Uppsala University, see Fig. 1 a). The device consists of a linear generator at the seabed connected to a surface buoy through a stiff wire; when the floating buoy oscillates, driven by the wave force, it drags the translator to move simultaneously. The surface of the translator is mounted with magnets and electric wires are wound inside the stator, implying that the translator motion will cause a changing magnetic field that generates voltage. It has been tested and refined in a long series of experiments at the research site outside Lysekil, the west coast of Sweden [1].

Many point absorbing wave energy concepts are based on pneumatic or hydraulic damping systems [2]. To eliminate the need for gear boxes and other sensitive and complex systems, direct-driven power take-off (PTO) systems were proposed as a viable alternative in [3], [4] and has been realized in several wave energy concepts [5]–[7], including the one developed at Uppsala University.
waves and buoy motions, the hydrodynamical forces will be computed using linear potential flow theory.

It has previously been shown, that the horizontal force has limited influence and the motion of a point absorber is dominated by the vertical force [22], [23]. For this reason, we now make the assumption of negligible horizontal motion of the buoy and focus on a heaving buoy, and elaborate on the methodology with this assumption.

For the WEC concept developed at Uppsala University, the connection line is non-rigid and can be stretched but not compressed. This implies that two situations may occur: if the connection line is tight, the motions of the buoy and the translator are coupled, otherwise they will move individually on their own. When the connection line is tight, the equations of motions are

\[ M_b \ddot{z}_b(t) = F_e(t) - F_r(t) - F_{\text{line}}(t) - M_b g + \rho g (V_{\text{sub}} - S z_b(t)), \]
\[ M_t \ddot{z}_t(t) = F_{\text{line}}(t) - F_{\text{endstop}}(t) - F_{\text{PTO}}(t) - M_t g, \]

which can be combined into one equation via the line force. Here the subscript \( b \) denotes the buoy, and \( t \) denotes the translator.

When the connection line is slack, the two motions become decoupled and the buoy floats freely with a decreased draft \( b' \). The corresponding excitation and radiation forces have to be calculated with respect to this reduced draft and are indicated as \( F_e' \) and \( F_r' \), and the equations of motion take the form

\[ M_b \ddot{z}_b(t) = F_e'(t) - F_r'(t) - \rho g S z_b(t), \]
\[ M_t \ddot{z}_t(t) = -F_{\text{endstop}}(t) - F_{\text{PTO}}(t) - M_t g. \]

In the decoupled case, the weight of the translator is the driving force for its motion. The equations of motion (1-4) describe the behaviour of a WEC system as a function of time.

The excitation force \( F_e \) is the force acting on a fixed buoy when it is subjected to the incident waves. The radiation force \( F_r \) is the force acting on an oscillating buoy when it is subjected to the radiated waves generated by itself. \( S \) is the cross-sectional area of the buoy. In the time domain, the excitation force is given as a convolution of the excitation force impulse response function \( f_e \) and the elevation of the incident waves \( \eta \).

\[ F_e(t) = f_e(t) * \eta(t). \]

The radiation force in the time domain can be written as:

\[ F_r(t) = m_a(\infty) \ddot{z}_b(t) + L(t) * \ddot{z}_b(t), \]

where \( m_a(\infty) \) is the constant value of the added mass at infinite frequency and \( L(t) \) the radiation impulse response function, which can be written in terms of either the added mass \( m_a \) or the radiation damping coefficient \( B \) as [24]

\[ L(t) = \frac{2}{\pi} \int_0^\infty \omega [m_a(\infty) - m_a(\omega)] \sin(\omega t) d\omega, \]
\[ L(t) = \frac{2}{\pi} \int_0^\infty B(\omega) \cos(\omega t) d\omega. \]

The connection line between the buoy and the translator can be modelled as a very stiff spring with a spring constant \( k_{\text{line}} \).
When $z_b < z_t$ the connection line is slack and the motion of the buoy and the translator become decoupled. A slack line occurs when the buoy falls faster than the translator, or when the translator is situated at the bottom of the generator, while the buoy is near or at the trough of a large amplitude wave. The frequency of the occurrence depends on both the wave climate and WEC configurations, for instance, a higher power take-off damping implies a slower motion of the translator and more events of slack line. The impulse force caused by the tightening of a slack line can be very large and may cause damage for the mechanical structure of a WEC unit. Knowing the instantaneous status of the connection line is one of the most important aspects when studying the survivability of the offshore device. Under the assumption of negligible line mass, the line force at the buoy and translator will be equal in magnitude as long as the line is not slack, with

$$ F_{\text{line}} = \begin{cases} k_{\text{line}}(z_b - z_t), & \text{if } z_b > z_t, \\ 0, & \text{else.} \end{cases} $$

(9)

$F_{\text{endstop}}$ is the force exerted by the compression of either the upper or lower end-stop springs,

$$ F_{\text{endstop}} = \begin{cases} k_u(z_t - l_u), & \text{if } l_u < z_t < l_u, \\ k_l(z_l + l_t), & \text{if } -l_t, \text{max} < z_t < -l_t, \\ 0, & \text{else.} \end{cases} $$

(10)

Here, the $l_u$ and $l_t$ are the the free stroke lengths that the translator is able to move upwards and downwards without compressing the end-stop springs, respectively. $l_u, \text{max}$ and $l_t, \text{max}$ are the maximum stroke lengths. Both the upper and lower end-stop springs are infinite in magnitude when $|z_l(t)|$ is equal to the maximum stroke lengths, preventing the translator to move any further.

The PTO damping force $F_{\text{PTO}}$ represents the electromagnetic damping force from the current in the stator windings which damp the motion of the translator. By taking into account that the area of the translator only partly overlaps with the area of the stator, as specified by the active area ratio $A_{\text{act}}$, for the resistive load, the PTO force can be written as

$$ F_{\text{PTO}}(t) = A_{\text{act}}(t)\gamma z_t(t), $$

(11)

where $A_{\text{act}}$ can be calculated by:

$$ A_{\text{act}}(t) = \begin{cases} 0, & |z_t| \geq (l_t + l_s)/2 \\ 1, & |z_t| \leq |l_t - \bar{l}_s|/2 \\ \frac{1}{l_t}((l_t + l_s)/2 - |z_t|), & \text{otherwise}, \end{cases} $$

(12)

where $l_t$ and $l_s$ are the lengths of the translator and stator.

### III. Method on deriving the circuit model

To derive an equivalent circuit model from the force equations, generally five steps are needed:

i. Converting each physical quantity in the force equation into an electrical component;

ii. Distinguishing if the electrical component has a constant or variable value;

iii. Identifying the layout of the electric circuit, deciding if it is parallel or series connected among components;

iv. Determining the direction when placing the electrical component, once it has positive and negative poles;

v. Adding switches in the electric circuit where the force equation changes its form, e.g., for piecewise functions.

In this paper, we follow the convention that the force is equivalent to the voltage drop; as all the force terms can be written in the form of Ohm’s law. The conversion relation has been summarized in Table I. The table states that a constant term in the force equation can be converted into a DC voltage source. Depending on which form the argument is — it could be $z$, $\dot{z}$ or $\ddot{z}$ in the force equations — the coefficient that is multiplied with it can be converted into a capacitor, resistor or inductor respectively. The velocity acts as current in the electric circuit on all occasions.

**Table I**

<table>
<thead>
<tr>
<th>Typical force</th>
<th>Circuit relation</th>
<th>Equivalence</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Mg$</td>
<td>$U_{DC} = \text{const.}$</td>
<td>[N] $\leftrightarrow$ [V]</td>
</tr>
<tr>
<td>$kz = (j\omega/k)^{-1} \ddot{z}$</td>
<td>$U_{DC} = j\omega C \gamma/t$</td>
<td>[m/N] $\leftrightarrow$ [F]</td>
</tr>
<tr>
<td>$\ddot{z}$</td>
<td>$U_R = R I$</td>
<td>$\gamma \Rightarrow R$</td>
</tr>
<tr>
<td>$M\ddot{z}$</td>
<td>$U_L = j\omega LI$</td>
<td>$M \leftrightarrow L$</td>
</tr>
<tr>
<td>$\dot{z}$</td>
<td>$\ddot{z}$</td>
<td>$\ddot{z} \Rightarrow \dddot{z}$</td>
</tr>
</tbody>
</table>

To eliminate the convolution term in the excitation force expression in the time domain, we assume the incident wave is a monochromatic harmonic wave $\eta(t) = \text{Re}(Ae^{i(\omega t + k z)})$ with wave amplitude $A$, wave number $k$ and angular frequency $\omega$. Then the excitation force is proportional to the incident wave $F_{\text{act}}(t) = \text{Re}(A f_{\text{ac}}(\omega)e^{i(\omega t + k z)})$. With the regular wave assumption, the excitation force is a harmonics with a clear analogue as a AC voltage source in the electric circuit.

In the frequency domain, the radiation force can be written as

$$ F_r(\omega) = [i\omega m_a(\omega) + B(\omega)]\ddot{z}_b(\omega), $$

(13)

which enables us to convert the added mass and the radiation damping coefficient to an inductor and resistor.

If the coefficient that is multiplied with the argument has a constant value, e.g., the spring constant, its related electrical parameter shall be constant as well. Naturally if this coefficient has a changing value, then the corresponding component’s parameter will be variable, e.g., the electromagnetic damping coefficient $\gamma$.

One equation of motion can be seen as counting the voltage drop over each component for one mesh, thus all terms in one equation are series connected in the circuit. For two equations of motion having the same force term, the remaining part of each equation will be in parallel connection with each other, and the equivalent electrical component for this force term will be a common component shared by the two meshes. One example is the force in connection line $F_{\text{line}}$ term in Eq. (1) and Eq. (2); its relevant capacitor $1/k_{\text{line}}$ is shared by two meshes, like Fig. 3 a) presented.

The force, displacement, velocity and acceleration are vectors and have directions. Formerly we have defined upwards as the positive direction for motion analysis, further we define clockwise as the positive direction for the circuit analysis, as indicated in Fig. 2 a). The positive voltage source will lead to
describes the case when the buoy line gets slack. Figure 3 a) refers to motion. In order to have a better classification of the numerous left is for the buoy motion, and the right is for the translator introduced point absorbing WEC system in Fig. 1 a) can PTO part to simulate the status that the translator would be compressed or not, etc. Those different situations would cause distinct force conditions, meaning different equations of motion need to be applied. Consequently, the equivalent circuit model would be different in layout in order to represent the new situation. The electric switch is the ideal electrical component to handle such cases. One example is drawn in Fig. 3: a single-pole triple-throw (SPTT) switch is placed in the PTO part to simulate the status that the translator would hit the upper/lower end-stop spring, or move freely.

figure
With help of the 5-step method introduced in this section, the introduced point absorbing WEC system in Fig. 1 a) can be modelled by an equivalent electric circuit that is presented in Fig. 3. The entire circuit model consists of two meshes, the left is for the buoy motion, and the right is for the translator motion. In order to have a better classification of the numerous parameters, the circuit model is divided by modules that are coloured with blue, yellow and grey. Figure 3 a) refers to the situation that the connection line is tight, and Fig. 3 b) describes the case when the buoy line gets slack.

IV. CIRCUIT MODEL FOR MORE CASES
The presented non-linear equivalent electric circuit model is based on a heaving buoy connected with a typical linear generator. This section will take three examples to illustrate how the equivalent electric circuit model is modified if the WEC designs are different from the presented one, or if the horizontal motion of the buoy is taken into consideration.

A. Generator with retracting springs
Some WEC designs consist of a retracting spring that is attached with the translator at the bottom of the generator, as Fig. 4 a) demonstrates.

The retracting spring force acts on the translator and can be decomposed into two parts: the constant preload force that only matters when the connection line is tight, and the varying spring force that is proportional to the displacement of the translator. At equilibrium position, \( pgV_{sub} = (M_b + M_t)g + F_{preload} \). The retracting spring force on the translator will be: \( F_s = k_z(z_t) + F_{preload} \). The dynamic equation of the translator is thereby rearranged as

\[
M_t \ddot{z}_t(t) = F_{line}(t) - F_{endstop}(t) - F_{PTO}(t) - M_t g - k_z(z_t(t) - F_{preload})
\]

When the connection line is slack, the preload force vanishes. The Archimedes force from the buoy only needs to balance the weight of itself, \( pgV_{sub} = M_bg \), and the retracting spring force on translator will be only the varying spring force. The translator motion equation becomes:

\[
M_t \ddot{z}_t(t) = -F_{endstop}(t) - F_{PTO}(t) - M_t g - k_z(z_t(t) \cdot (15))
\]

Figure 4 b) draws the equivalent electric circuit of this case for the tight connection line status. It is evident that the varying spring force \( k_z(z_t(t) \) is equivalent to the voltage drop over a capacitor, and the preload force \( F_{preload} \) is equivalent to a DC voltage source. For the slack connection line status, the preload DC voltage source will disappear.

The retracting spring and its corresponding electrical components are highlighted in violet in Fig. 4. To emphasise the difference brought in by the new element, we use black fonts to illustrate the altered part, and grey fonts for the ones that remain the same. In this example, all three modules get influenced by the retracting spring. For the wave-buoy interaction part, the hydrodynamic force would be different in value because of the different draft. For the buoy configuration, the draft will be changed. And for the PTO module, the preload force and the varying spring force terms will be introduced in.

B. Rubber damper for a better survivability
To prevent the generator and connection line from disruptive forces during large waves, aside from the end-stop springs, a rubber damper can be installed in the connection line for a better survivability of the WEC unit. Ref. [25] has conducted an analytical study on implementation of the rubber damper in a WEC unit.

The rubber damper structure is normally placed in the center of the buoy. It behaves as a spring-damper system, which contains a viscous damper with damping coefficient \( C_{damper} \), and a spring with spring constant \( k_{damper} \), as Fig. 5 a) illustrates.

If taking the existed steel wire and the rubber damper as the new 'connection line system', perceivably the rubber damper would be subjected to the same magnitude of the line force as the steel wire. Thanks to the contribution of the viscous damping and the elasticity of the rubber damper, the entire 'connection line' system response to a snatch load will change. To be more specific, a different distribution of the total displacement difference \( |z_b - z_t| \) at big waves can be expected: it equals the compressed length of the rubber damper \( \Delta z_{damper} \), plus the elongated length of the steel wire \( \Delta z_{line} \). In comparison to the former design when all the displacement difference exerted on the steel wire which would cause a breaking force, the new design will ease the line force by sharing the material deformation between the steel wire...
gets installed is presented in Fig. 5 b), the rubber damper and its relevant electrical components are highlighted in green. The elastic force from deformation of the steel wire and the rubber damper can be seen as the voltage drop over a capacitor, and the viscous damping coefficient is analogous to resistance in the electric circuit. Eq. (16) reveals that the resistor $C_{\text{damper}}$ is in series connection with capacitor $1/k_{\text{damper}}$, and they shall be in parallel connection with capacitor $1/k_{\text{line}}$. The slack connection line status simply corresponds to a disconnection status for the connection line branch, its equivalent circuit is the same as Fig. 3 b).

### C. Coupled heave and surge motion

Considering that the horizontal force may differ for different buoy shapes, and that a small horizontal motion may become a criteria when it comes to buoy optimization, we here also establish an equivalent electric circuit model for the situation where the buoy moves in both heave and surge, as depicted in Fig. 6. The surge motion part is highlighted in orange. The

and the rubber damper. The line force and the displacement relation in this case can be written as:

\[
F_{\text{line}} = \begin{cases} 
    k_{\text{line}} \Delta z_{\text{line}}(t) = k_{\text{damper}} \Delta z_{\text{damper}}(t) + C_{\text{damper}} \Delta \dot{z}_{\text{damper}}(t), & \text{if } z_b > z_t \\
    0, & \text{else.}
\end{cases}
\]

\[
|z_b - z_t| = \Delta \dot{z}_{\text{damper}} + \Delta z_{\text{line}}.
\]

The equivalent electric circuit model when the rubber damper

motion of the buoy in two degrees of freedom and the vertical motion of the translator will be coupled by the line force. Denote the line force lifting the translator as $F_{\text{line,t}}$ in this case, to distinguish from the line force at the buoy $F_{\text{line,b}}$ =
For a tight connection line, the line force at the buoy and at the translator are equal in magnitude.

Denote the constant distance between the free surface and the connection point of the translator by \( l \). The displacement of the buoy in the \( x \)-direction relative to the \( z \)-direction can be described in terms of a parameter \( \varepsilon(t) = \frac{z_i(t)}{\sqrt{1 + (t^2)^2}} = \tan \alpha(t) \), where \( \alpha(t) \) is the polar angle of the buoy displacement.

As long as the buoy line is not slack, the vertical displacement of the translator is related to the position of the buoy as

\[
z_i(t) = (1 + z_b(t))\sqrt{1 + \varepsilon(t)^2} - l,
\]
and the line force at the buoy in the \( x \)-direction can be written as \( F_{\text{line},b} = \varepsilon F_{\text{line},b} \) so that

\[
F_{\text{line},t} = \sqrt{F_{\text{line},b}^2 + F_{\text{line},z}^2} = \sqrt{1 + \varepsilon^2} F_{\text{line},b}.
\]

The equations of motion for the buoy can be written as

\[
\begin{align*}
M_b \ddot{z}_b(t) &= F_{r,x}(t) - F_{r,z}(t) - \varepsilon(t) F_{\text{line},b}(t), \\
M_b \ddot{\theta}_b(t) &= F_{r,z}(t) - F_{\text{line},b}(t) - M_b g, \\
&+ pg(V_{\text{sub}} - S \dot{z}_b(t))
\end{align*}
\]

For an axisymmetric cylinder, the heave and surge motions are hydrodynamically decoupled, and independent radiation forces in surge and heave on the form as in Eq. (6) are obtained,

\[
\begin{align*}
F_{r,x}(t) &= m_a(\infty) \ddot{x}_b(t) + L_{11}(t) \dot{x}_b(t), \\
F_{r,z}(t) &= m_a(\infty) \ddot{z}_b(t) + L_{33}(t) \dot{z}_b(t).
\end{align*}
\]

Since the translator is constrained to vertical motion by the funnel design, the equation of motion for the translator is the same as the case when the buoy moves in heave only, as in Eq. (2). Together with the relation between the line force at the buoy and the translator in Eq. (19), the equation of motion for the translator is then,

\[
M_t \ddot{z}_t(t) = \sqrt{1 + \varepsilon(t)^2} F_{\text{line},b}(t) - F_{\text{PTO}}(t) - F_{\text{endstop}}(t) - M_t g.
\]

The full system is now described by the coupled equations of motion (20) and (22).

To establish the equivalent circuit model with consideration of the surge motion of the buoy, we can start from the motion equation for the tight line situation. The line forces \( F_{\text{line},b} \) and \( F_{\text{line},z} \) are orthogonal, which corresponds to a 90° phase shift in the circuit diagram. The heave and surge motions of the buoy are interrelated with each other by the line force, as in Eq. (19). This implies that the two components that supply the line force are connected in series, and there is a 90° phase shift between the voltage drop over them. Therefore, one can model the line force in \( x \)-direction as equivalent to the voltage drop over a resistor, and in \( z \)-direction as equivalent to the voltage drop over a capacitor, as Fig. 6 b) demonstrates. The expression of the resistance \( \varepsilon k_{\text{line}} / \omega \) is derived based on the relation that the line force in the heave direction is \( |F_{\text{line},b}| = k_{\text{line}} \Delta z \), while in the surge direction it is \( |F_{\text{line},b}| = \varepsilon |F_{\text{line},b}| \).

Since the heave and surge motions of the buoy are independent; if there is no wave-buoy interaction in heave, then in the electric circuit model, the heave branch is an open circuit, and the surge branch has formed a loop of its own. Likewise, if there is no wave-buoy interaction in surge, the surge branch that is in parallel connection with the heave branch will be an open circuit. Simultaneously, \( \varepsilon = 0 \), which means the resistance of \( \varepsilon k_{\text{line}} / \omega \) goes to zero — it acts as an electric wire for the AC circuit, implying that the electric circuit becomes equivalent to the case with heave motion.

V. APPLICATION OF THE CIRCUIT MODEL

Two types of the WEC system analysis can be carried out if using Matlab Simulink to simulate the proposed equivalent circuit model: the transient analysis and the analysis with respect to a long-term wave climate. In the simulations, the dynamics of the buoy and the translator is computed at each time instant. The result is taken as the input for the switches, and the circuit automatically changes between different states. Theoretically, all the physical quantities in the motion equations, like displacement, velocity or force, as well as quantities that can be derived based upon those, such as power, can be simulated and read from the circuit model simulation. The expected analysis results for two types of the studies are described below, the WEC parameters for the simulation are listed in Table II.

| Buoy radius | 1.5 m |
| Buoy height | 0.8 m |
| \( b \) | 0.4 m |
| \( b' \) | 0.14 m |
| \( M_b \) | 1000 kg |
| \( M_t \) | 1000 kg |
| \( k_a \) | 6.2 kN/m |
| \( \gamma \) | 56 kN/m |
| \( k_{\text{line}} \) | 450 kN/m |
| \( k_i \) | 215 kN/m |
| \( l_u \) | 0.895 m |
| \( l_i \) | 0.895 m |
| \( l_{\text{max}} \) | 1.1 m |

TABLE II

WEC PARAMETERS USED FOR SIMULATIONS.

A. Study 1: transient analysis

The introduced methodology can be used to investigate the transient response for one specific wave which is characterised by a wave period \( T \) and wave height \( H \). Figure 7 illustrates what outputs can be expected from the study. In this example, two incident waves are tested, one is a moderate wave with wave period of 5 s and wave height of 1.5 m, the other is a bigger wave with wave period of 6 s and wave height of 2.5 m. The results from the moderate wave are drawn in the thin linewidth, while for the bigger wave the linewidth is thicker.

The instantaneous value of the force in the connection line, and the velocity can be displayed in a scope in Simulink. In Fig. 7 c) and d), curves for the buoy are in black and are in orange for the translator. The absorbed power can be obtained by placing a multiplication block in the circuit to multiply the PTO force with velocity of the translator. The displacement can be obtained by adding an integrator block after the velocity signal in the circuit, and the saturation limit in the integrator block can be defined to mimic the maximum stroke length of the translator. Basically all the quantities would like to know can be read directly in a scope from the circuit simulation.

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B. Study 2: an overall evaluation over a long period

To evaluate a WEC system in realistic conditions, it is important to study its performance over a long time period with wave data collected at an offshore site. For such a long time system evaluation, the velocity and displacement of the buoy and the translator become less critical, while an overall time system evaluation, the velocity and displacement of the buoy has an initial surge motion, it will be subjected to the radiation force only, and the motion will be damped and vanish eventually. The superposition method will lead to a result that in heave and surge’ case even though two voltage sources are independent. The reason is that the heave excitation force will reflect the physical status of the WEC system, therefore the principle of superposition, can not be used in the ‘buoy moves in heave and surge’ case even though two voltage sources are independent, here we will further discuss four aspects related to the circuit modelling method.

VI. DISCUSSION

This paper has presented the methodology of deriving an equivalent circuit model for point absorbing WEC systems. The key is to link a physical force equation to the electric circuit theory. To ensure that the model can be created correctly and used smoothly, here we will further discuss four aspects related to the circuit modelling method.

A. Links between the WEC system to the electric circuit

The electric circuit model can be seen as a media that reflects the physical status of the WEC system, therefore the analysis of the electric circuit always has to be considered together with the force analysis: techniques to analyse the electric circuit can not be applied directly unless it makes sense in reality. Ohm’s law is the starting point to determine the conversion relation, it is always valid for the equivalent circuit model analysis. The mesh analysis can be used to check if the force equation for the buoy or the translator is consistent with the Kirchhoff’s voltage law (KVL).

However, some circuit analysis techniques, such as the principle of superposition, can not be used in the ‘buoy moves in heave and surge’ case even though two voltage sources are independent. The reason is that the heave excitation force will not result in a radiation force in surge, and vice versa. If a buoy has an initial surge motion, it will be subjected to the radiation force only, and the motion will be damped and vanish eventually. The superposition method will lead to a result that even if there is no voltage source in the surge motion branch, the current from the heave motion branch will supply both the PTO part and the impedance in the surge motion branch, which conflicts with the motion analysis. Therefore if there is no motion in a certain direction, the correct way is to disconnect the branch that represents the motion in this direction.
B. Modular thinking makes it easier

Modular thinking has been applied when deriving the equivalent circuit models for different cases. It makes the circuit model more structured and easier to be modified. After identifying which part of the dynamic WEC system would be influenced by the change — could be the wave-buoy hydrodynamic interaction, the configuration of the buoy, the connection line or the PTO part — the corresponding modules in the equivalent circuit model can be modified. Figure 9 presents the circuit model that contains all the three cases in Section IV. Building this electric circuit without the modular approach would soon get quite involved.

![Diagram](image_url)  
Fig. 9. The equivalent electric circuit model for a WEC unit whose buoy is moving in both heave and surge directions. The connection line is equipped with a rubber damper and a retracting spring is installed in the generator.

The dashed line in the electric circuit model entails the three switches will act simultaneously, since technically they occur at the same time for the buoy and the translator. The status shown in the figure is when the line is slack. If $S_{1-3}$ switch to the right side, it is the status that the line is tight. The switch $S_4$ that controls whether the end-stop springs get compressed is an independent switch.

C. Beyond the circuit simulation

The elaborated two examples in Section V have covered both the micro and macro dynamic system analysis for a WEC system. The transient study provides a tool to study the details of the system response to one specific incident wave, e.g., phase shift between two quantities. It could in particular be useful when examining the WEC system performance in the dominate wave climate.

The sweep of the power and force over a matrix of wave climate provides a tool for the estimation of the electrical energy production and the peak force distribution. It can be used for the optimization of a WEC unit for one specific site. The configuration of the WEC, e.g., translator weight, can be adjusted to search for the solution that can produce the most electrical energy while the line force is not so large.

D. Advantages and disadvantages of the method

Comparing with conventional modelling methods, the presented equivalent RLC circuit model provides a widely known tool for engineers with different backgrounds, which is desired for the multidisciplinary characterized WEC system modelling. Besides, the equivalent circuit model makes the elements in a WEC system more visualized. Major parameters that may influence the power production of a WEC system are exhibited in a one-layer circuit network, there is no hidden equations behind the electrical components, nor sub-systems in the simulation model, which implies that the model can be easily interpreted and used. The dynamic interactions in the wave-buoy-PTO system can be simulated in one interface, which has simplified the system modelling process. It is easy to evaluate different PTO configurations efficiently for generator design or electrical system studies, and optimized values of design parameters in different wave climates can be found.

In addition, the calculation complexity of solving the motion equation in time domain has been largely simplified. Due to the fact that the circuit takes monochromatic harmonic waves as inputs, the convolution term $f(t) \ast g(t)$ of the excitation force is no longer present, and the frequency decided coefficients including added mass $m_a$ and radiation damping coefficient $B$ become constant numbers. Consequently, the computational time is reduced.

However, the limitation of the current circuit model is obvious as well: the analysis in this paper is valid for regular waves only. This simplified setting serves as a feasible foundation for the aspects we wish to study, but the results should be regarded with some caution due to the different behaviour of WEC systems in regular waves as compared to the dynamics in realistic, irregular waves [19]. Admittedly, the linear assumption is questionable if the WEC device is operating in large seas or is undergoing large amplitude oscillations. The sensitivity to nonlinear effects by the use of a hierarchical robust controller was demonstrated in [10]. Moreover, the proposed method currently relies on external software to compute the hydrodynamics and FEM-analysis of the generator, which puts a limit to its usage. This can be overcome by integrating the circuit with analytical and numerical codes for hydrodynamics and generator simulations.

VII. Conclusion

This paper elaborates on the methodology of establishing a non-linear equivalent electric circuit model to be used as a system modelling tool for point absorbing wave energy converter systems. Three increasingly complex examples are supplied to illustrate the methodology. The suggested equivalent circuit model provides an integrated interface which is considered useful for both transient analysis and long term evaluation of the system performance. It allows rapid assessments on some key performance indicators such as force in the connection line, absorbed power, etc. In addition, the total electrical energy production over certain periods can be estimated. This fast modelling method will facilitate the design, adjustment, and optimization for certain major parameters as well as the entire wave power system.
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REFERENCES


